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Magnetic susceptibility of QCD vacuum at finite density from the nonlocal chiral quark model

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Abstract

We present in this talk a recent investigation on the magnetic susceptibility (χ) of the QCD vacuum at finite density, utilizing the nonlocal chiral quark model from the instanton vacuum. We take into account the nonzero current-quark mass (m_q) explicitly to consider the effect of explicit flavor SU(3) symmetry breaking. It turns out that, when we turn on the current-quark mass ($m_q \neq 0$), χ becomes smaller, indicating less response to the externally induced electromagnetic field, in comparison to that for $m_q = 0$.

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I. INTRODUCTION

Properties of hadrons are known to be changed at finite density (μ) and/or temperature (T). Thus, it is essential to understand the mechanism of the modification of haronic properties in medium, based on quantum chromodynamics (QCD) which is the underlying theory of the strong interaction. The first step to describe hadrons at finite density is to investigate the in-medium modification of the QCD vacuum.

In the present talk, we want to study the magnetic susceptibility of the QCD vacuum at the finite quark (number) chemical potential (μ_q) at $T=0$, which is defined as

$$\langle iq^\dagger \sigma_{\mu\nu} q\rangle_F = ie_q F_{\mu\nu}\langle iq^\dagger q\rangle\chi, \tag{1}$$

where $\langle iq^\dagger q\rangle$ stands for the chiral condensate. In principle, the magnetic susceptibility represents the response of the vacuum in the presence of the external electromagnetic (EM) field. Moreover, the present study may shed light on the interpretation for the magnetic properties of the compact star from the hadronic origin [1].

Here, we restrict ourselves to the spontaneous chiral symmetry breaking (SχSB) phase. We also take into account the nonzero current-quark mass [2, 3, 4]. We observe that, due to the competition between m_q and μ_q, the finite current-quark mass changes the magnetic susceptibility drastically, in particular, in the region of higher chemical potentials, compared to the case of the chiral limit: The χ_μ for the massless quark increases stiffly whereas it is relatively flat or decreases with the finite current-quark mass.

II. NONLOCAL CHIRAL QUARK MODEL AT FINITE DENSITY

We start with the quark zero-mode equation with the chemical potential in the (anti)instanton effects in Euclidean space:

$$[i\partial\!\!\!/ - i\mu\!\!\!/ + im_q + A\!\!\!/_{I\bar{I}}]\,\Psi^{(0)}_{I\bar{I}} = 0. \tag{2}$$

Note that the quark chemical potential four vector $\mu = (0,0,0,\mu_q)$ and current-quark mass m_q are explicitly included in the operator. $A_{\bar{I}I}$ stands for the (anti)instanton contribution whereas the $\Psi^{(0)}_{I\bar{I}}$ denotes the quark zero-mode solution. From this, the effective quark

propagator in the (anti)instanton effects can be derived as follows:

$$S=\frac{1}{i\partial\!\!\!/-i\mu\!\!\!/+im_q+i\mathcal{M}},\quad \mathcal{M}\approx\hat{\mathcal{M}}(i\partial,\mu)\left[\sqrt{1+\frac{m_q^2}{d}}-\frac{m_q}{d}\right], \tag{3}$$

where $\mathcal{M}(i\partial,\mu)$ denotes the momentum-dependent quark mass, which arises from the Fourier transformed zero-mode solution of Eq. (2), and its explicit form is given in Ref. [5]. The parameter d is chosen to be the vacuum value, 198 MeV for simplicity. As for the effective quark mass, we, however, parameterize it in the following form:

$$\hat{\mathcal{M}}(i\partial,\mu)\;=\;\mathcal{M}_0\left[\frac{2\Lambda^2}{(i\partial-i\mu)(i\partial-i\mu)+2\Lambda^2}\right]^2=\mathcal{M}_0\mathcal{F}^2(i\partial,\mu), \tag{4}$$

for simplicity. Note that this dynamical quark mass is complex in general for $\mu_q\neq 0$. Here we use $\Lambda\approx 600$ MeV and $\mathcal{M}_0\approx 350$ MeV in connection to the instanton packing fraction at vacuum, $(N/V)^{-4}\approx 200$ MeV. Note that this parameterization works well in relatively dilute region, $\mu_{\rm ch.}\lesssim 200$ MeV.

Now we are in a position to consider the vacuum expectation value (VEV) shown in Eq. (1) with the modified effective action, derived from the instanton vacuum in the presence of the μ and EM field:

$$\mathcal{S}_{\rm eff}[\mu,m_q,\mathcal{T}]=-{\rm Sp}\ln\left[iD\!\!\!\!/-i\mu\!\!\!/+im_q+i\mathcal{M}(iD,\mu,m_q)+\sigma\cdot\mathcal{T}\right], \tag{5}$$

where Sp is the functional trace $\int d^4x\langle x|\cdots|x\rangle$ running over Dirac and color indices. The iD indicates the covariant derivative, $i\partial+e_qA$, in which e_q and A are the quark electric charge and externally induced photon field, respectively. $\mathcal{T}$ denotes the external tensor field. Performing the functional differentiation with respect to $\mathcal{T}$, we obtain the following expression for the VEV of Eq. (1) in a operator form:

$$-N_c{\rm tr}_\gamma\left\{\left[\frac{iD\!\!\!\!/-i\mu\!\!\!/-i\bar{\mathcal{M}}_F}{(i\partial-i\mu)^2+\frac{\sigma\cdot F}{2}-[iD\!\!\!\!/,\mathcal{M}_F]+\bar{\mathcal{M}}_F^2}-\frac{iD\!\!\!\!/-i\mu\!\!\!/-im_q}{-\partial^2+\frac{\sigma\cdot F}{2}+m_q^2}\right]\sigma_{\mu\nu}\right\}, \tag{6}$$

where we use the abbreviation $\bar{\mathcal{M}}_F=\mathcal{M}(iD,\mu_q,m_q)+m_q$ for convenience. $F_{\mu\nu}$ is the field strength tensor for the external photon field. ${\rm tr}_\gamma$ denotes the trace over spin space. Finally we arrive at the expression for the $\chi_\mu\langle iq^\dagger q\rangle_\mu$ as follows:

$$4N_c\int_\infty^\infty\frac{d^4p}{(2\pi)^4}\left[\frac{\bar{\mathcal{M}}}{[(p+i\mu)^2+\bar{\mathcal{M}}^2]^2}-\frac{m_q}{[p^2+m_q^2]^2}-\frac{\mathcal{M}'\cdot(p+i\mu)}{[(p+i\mu)^2+\bar{\mathcal{M}}^2]^2}\right], \tag{7}$$

where $\mathcal{M}=\mathcal{M}(p,\mu,m_q)$. Here $\mathcal{M}'$ represents the derivative of $\mathcal{M}$ with respect to p.

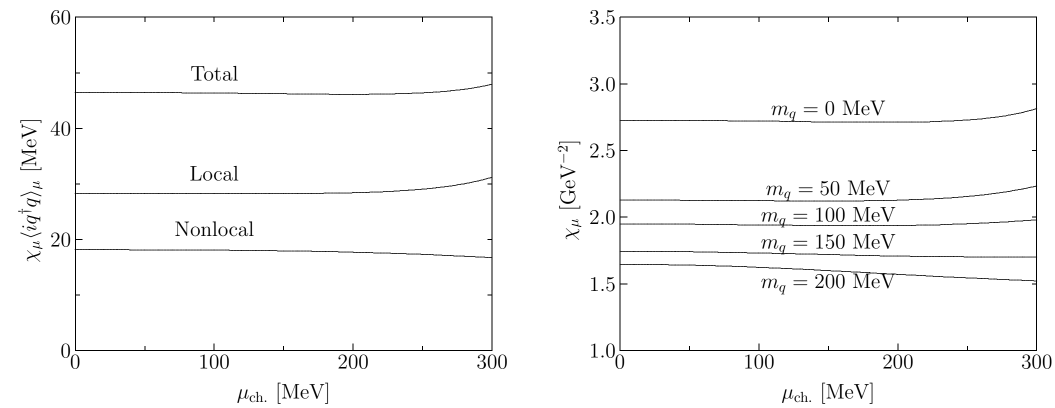

FIG. 1: Left panel: $\chi_\mu\langle iq^\dagger q\rangle_\mu$ for each contribution. Right panel: χ_μ for different values of the current-quark mass, m_q.

III. NUMERICAL RESULTS

First, those for the $\chi_\mu\langle iq^\dagger q\rangle_\mu$ are given in the left panel of Fig. 1 in which we draw it in the chiral limit for each contribution separately. We draw them up to $\mu_{\rm ch.} \approx 300$ MeV. Note that its vacuum value becomes about 46.6 MeV, which is consistent with that computed in Ref. [6]. This value is also compatible to that from the QCD sum rules [7, 8, 9]. While the nonlocal contributions decrease slightly with respect to the chemical potential, the local one increases slowly. This difference is due to the different behaviors of $\mathcal{M}$ and $\mathcal{M}'$ for the μ_q.

In the right panel of Fig. 1, we depict the χ_μ for different values of the m_q. We note that the QCD magnetic susceptibilities computed are very stable with respect to $\mu_{\rm ch.}$. When we turn on the current-quark mass ($m_q \neq 0$), χ_μ becomes smaller, indicating less response to the externally induced EM field.

IV. SUMMARY AND CONCLUSION

In the present talk, we have presented the magnetic susceptibility (χ_μ) of the QCD vacuum for the nonzero quark number chemical potential, $\mu_q \neq 0$, based on the nonlocal chiral quark model for $N_f = 1$, derived from the instanton vacuum in the presence of the chemical potential and external EM field. We also have considered the effect of flavor SU(3) symmetry breaking. We found that the QCD magnetic susceptibilities are in general very stable with respect to the chemical potential up to about 300 MeV. When we turn on the

current-quark mass, the magnetic susceptibility becomes smaller, in comparison to that for $m_q = 0$.

We conclude from the present results that the magnetic susceptibility of the QCD vacuum at finite density depends much on the current-quark mass. These interesting observations may shed light on the extreme phenomena taking place in the heavy-ion collision and compact stars, in particular, associated with heavier quarks such as the strange quark. A detailed investigation is under progress and will appear elsewhere.

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